21.2 Applications of Electric Fields

As you have learned, the concept of energy is extremely useful in mechanics. The law of conservation of energy allows us to solve motion problems without knowing the forces in detail. The same is true in the study of electrical interactions. The work performed moving a charged particle in an electric field can result in the particle’s gaining potential, or kinetic energy, or both. Because this chapter investigates charges at rest, only changes in potential energy will be discussed.

Energy and Electric Potential

Recall the change in gravitational potential energy of a ball when it is lifted, as shown in Figure 21-5. Both the gravitational force, $F$, and the gravitational field, $g = F/m$, point toward Earth. If you lift a ball against the force of gravity, you do work on it, thereby increasing its potential energy.

The situation is similar with two unlike charges: they attract each other, and so you must do work to pull one charge away from the other. When you do the work, you transfer energy to the charge where that energy is stored as potential energy. The larger the test charge, the greater the increase in its potential energy, $\Delta PE$.

Although the force on the test charge depends on its magnitude, $q'$, the electric field it experiences does not. The electric field, $E = F/q'$, is the force per unit charge. In a similar way, the electric potential difference, $\Delta V$, is defined as the work done moving a positive test charge between two points in an electric field divided by the magnitude of the test charge.

\[
\text{Electric Potential Difference} \quad \Delta V = \frac{W_{\text{on } q'}}{q'}
\]

The difference in electrical potential is the ratio of the work needed to move a charge to the strength of that charge.

Electric potential difference is measured in joules per coulomb. One joule per coulomb is called a volt (J/C = V).

Consider the situation shown in Figure 21-6 on the next page. The negative charge creates an electric field toward itself. Suppose you place a small positive test charge in the field at position A. It will experience a force in the direction of the field. If you now move the test charge away from the negative charge to position B, as in Figure 21-6a, you will have to exert a force, $F$, on the charge. Because the force that you exert is in the same direction as the displacement, the work that you do on the test charge is positive. Therefore, there also will be a positive change in the electric potential difference. The change in potential difference does not depend on the magnitude of the test charge. It depends only on the field and the displacement.
Electric potential difference is determined by measuring the work per unit charge. If you move unlike charges apart, you increase the electric potential difference (a). If you move unlike charges closer together, you reduce the electric potential difference (b).

Suppose you now move the test charge back to position A from position B, as in Figure 21-6b. The force that you exert is now in the direction opposite the displacement, so the work that you do is negative. The electric potential difference is also negative. In fact, it is equal and opposite to the potential difference for the move from position A to position B. The electric potential difference does not depend on the path used to go from one position to another. It does depend on the two positions.

Is there always an electric potential difference between the two positions? Suppose you move the test charge in a circle around the negative charge. The force that the electric field exerts on the test charge is always perpendicular to the direction in which you moved it, so you do no work. Therefore, the electric potential difference is zero. Whenever the electric potential difference between two or more positions is zero, those positions are said to be at equipotential.

Only differences in potential energy can be measured. The same is true of electric potential; thus, only differences in electric potential are important. The electric potential difference from point A to point B is defined as \( \Delta V = V_B - V_A \). Electric potential differences are measured with a voltmeter. Sometimes, the electric potential difference is simply called the voltage. Do not confuse electric potential difference, \( \Delta V \), with the unit for volts, V.
You have seen that electric potential difference increases as a positive test charge is separated from a negative charge. What happens when a positive test charge is separated from a positive charge? There is a repulsive force between these two charges. Potential energy decreases as the two charges are moved farther apart. Therefore, the electric potential is smaller at points farther from the positive charge, as shown in Figure 21-7.

As you learned in Chapter 11, the potential energy of a system can be defined as zero at any reference point. In the same way, the electric potential of any point can be defined as zero. No matter what reference point is chosen, the value of the electric potential difference from point A to point B always will be the same.

**The Electric Potential in a Uniform Field**

A uniform electric force and field can be made by placing two large, flat, conducting plates parallel to each other. One is charged positively and the other is charged negatively. The electric field between the plates is constant, except at the edges of the plates, and its direction is from the positive to the negative plate. The pattern formed by the grass seeds pictured in Figure 21-8 represents the electric field between parallel plates.

If a positive test charge, $q'$, is moved a distance, $d$, in the direction opposite the electric field direction, the work done is found by the relationship $W_{on q'} = Fd$. Thus, the electric potential difference, the work done per unit charge, is $\Delta V = Fd/q' = (F/q')d$. Now, the electric field intensity is the force per unit charge, $E = F/q'$. Therefore, the electric potential difference, $\Delta V$, between two points a distance, $d$, apart in a uniform field, $E$, is represented by the following equation.

**Electric Potential Difference in a Uniform Field**

$\Delta V = Ed$

The electrical potential difference in a uniform field is equal to the product of electric field intensity and the distance moved by a charge.

The electric potential increases in the direction opposite the electric field direction. That is, the electric potential is higher near the positively charged plate. By dimensional analysis, the product of the units of $E$ and $d$ is (N/C)(m). This is equivalent to one J/C, which is the definition of 1 V.
21. What work is done when 3.0 C is moved through an electric potential difference of 1.5 V?

22. A 12-V car battery can store $1.44 \times 10^6$ C when it is fully charged. How much work can be done by this battery before it needs recharging?

23. An electron in a television picture tube passes through a potential difference of 18,000 V. How much work is done on the electron as it passes through that potential difference?

24. If the potential difference in problem 18 is between two parallel plates that are 2.4 cm apart, what is the magnitude of the electric field between them?

25. The electric field in a particle-accelerator machine is $4.5 \times 10^5$ N/C. How much work is done to move a proton 25 cm through that field?
Millikan’s Oil-Drop Experiment

One important application of the uniform electric field between two parallel plates is the measurement of the charge of an electron. This first was determined by American physicist Robert A. Millikan in 1909. Figure 21-9 shows the method used by Millikan to measure the charge carried by a single electron. First, fine oil drops were sprayed from an atomizer into the air. These drops were charged by friction with the atomizer as they were sprayed. Gravity acting on the drops caused them to fall, and a few of them entered the hole in the top plate of the apparatus. An electric potential difference then was placed across the two plates. The resulting electric field between the plates exerted a force on the charged drops. When the top plate was made positive enough, the electric force caused negatively charged drops to rise. The electric potential difference between the plates was adjusted to suspend a charged drop between the plates. At this point, the downward force of Earth’s gravitational field and the upward force of the electric field were equal in magnitude.

The magnitude of the electric field, \( E \), was determined from the electric potential difference between the plates. A second measurement had to be made to find the weight of the drop using the relationship \( mg \), which was too tiny to measure by ordinary methods. To make this measurement, a drop first was suspended. Then, the electric field was turned off, and the rate of the fall of the drop was measured. Because of friction with the air molecules, the oil drop quickly reached terminal velocity, which was related to the mass of the drop by a complex equation. Using the measured terminal velocity to calculate \( mg \) and knowing \( E \), the charge, \( q \), could be calculated.

Charge on an electron Millikan found that there was a great deal of variation in the charges of the drops. When he used X rays to ionize the air and add or remove electrons from the drops, he noted, however, that the changes in the charge on the drops were always a multiple of \( 1.60 \times 10^{-19} \) C. The changes were caused by one or more electrons being added to or removed from the drops. Millikan concluded that the smallest change in charge that could occur was the amount of charge of one electron. Therefore, Millikan proposed that each electron always has the same charge, \( 1.60 \times 10^{-19} \) C. Millikan’s experiment showed that charge is quantized. This means that an object can have only a charge with a magnitude that is some integral multiple of the charge of an electron.

Millikan’s Oil-Drop Experiment

Electric Fields

Tie a pith ball on the end of a 20-cm nylon thread and tie the other end to a plastic straw. Holding the straw horizontally, notice that the ball hangs straight down. Now rub a piece of wool on a 30 cm \( \times \) 30 cm square of plastic foam to charge both objects. Stand the foam vertically. Hold the straw and touch the pith ball to the wool.

1. Predict what will happen when the ball is close to the foam.
2. Test your prediction by slowly bringing the hanging ball toward the charged plastic foam.
3. Predict the ball’s behavior at different locations around the foam, and test your prediction.
4. Observe the angle of the thread as you move the pith ball to different regions around the foam.

Analyze and Conclude

5. Explain, in terms of the electric field, why the ball swings toward the charged plastic.
6. Compare the angle of the thread at various points around the foam. Why did it change?
7. Infer what the angle of the thread indicates about the strength and the direction of the electric field.

Millikan’s experiment showed that charge is quantized. This means that an object can have only a charge with a magnitude that is some integral multiple of the charge of an electron.
Finding the Charge on an Oil Drop  In a Millikan oil-drop experiment, a drop has been found to weigh $2.4 \times 10^{-14}$ N. The parallel plates are separated by a distance of 1.2 cm. When the potential difference between the plates is 450 V, the drop is suspended, motionless.

a. What is the charge on the oil drop?

b. If the upper plate is positive, how many excess electrons are on the oil drop?

1 Analyze and Sketch the Problem

- Draw the plates with the oil drop suspended between them.
- Draw and label vectors representing the forces.
- Indicate the potential difference and the distance between the plates.

2 Solve for the Unknown

To be suspended, the electric force and gravitational force must be balanced.

\[ \frac{q \Delta V}{d} = F_g \]

Solve for \( q \).

\[ q = \frac{F_g d}{\Delta V} \]

\[ = \frac{(2.4 \times 10^{-14} \text{ N})(0.012 \text{ m})}{450 \text{ V}} \]

\[ = 6.4 \times 10^{-19} \text{ C} \]

Solve for the number of electrons on the drop.

\[ n = \frac{q}{e} \]

\[ = \frac{6.4 \times 10^{-19} \text{ C}}{1.6 \times 10^{-19} \text{ C}} \]

\[ = 4 \]

3 Evaluate the Answer

- Are the units correct? \( N \cdot m/V = J/(J/C) = C \), the unit for charge.
- Is the magnitude realistic? This is a small whole number of elementary charges.

26. A drop is falling in a Millikan oil-drop apparatus with no electric field. What forces are acting on the oil drop, regardless of its acceleration? If the drop is falling at a constant velocity, describe the forces acting on it.

27. An oil drop weighs $1.9 \times 10^{-15}$ N. It is suspended in an electric field of $6.0 \times 10^3$ N/C. What is the charge on the drop? How many excess electrons does it carry?

28. An oil drop carries one excess electron and weighs $6.4 \times 10^{-15}$ N. What electric field strength is required to suspend the drop so it is motionless?

29. A positively charged oil drop weighing $1.2 \times 10^{-14}$ N is suspended between parallel plates separated by 0.64 cm. The potential difference between the plates is 240 V. What is the charge on the drop? How many electrons is the drop missing?
Sharing of Charge

All systems come to equilibrium when the energy of the system is at a minimum. For example, if a ball is placed on a hill, it finally will come to rest in a valley where its gravitational potential energy is smallest. This also would be the location where its gravitational potential has been reduced by the largest amount. This same principle explains what happens when an insulated, positively charged metal sphere, such as the one shown in Figure 21-10, touches a second, uncharged sphere. The excess charges on sphere A repel each other, so when the neutral sphere, B, touches sphere A, there is a net force on the charges on A toward B. Suppose that you were to physically move the charges, individually, from A to B. When you move the first charge, the other charges on A would push it toward B, so to control its speed, you would have to exert a force in the opposite direction. Therefore, you do negative work on it, and the electric potential difference from A to B is negative. When the next few charges are moved, they feel a small repulsive force from the charges already on B, but there is still a net positive force in that direction. At some point, the force pushing a charge off A will equal the repulsive force from the charges on B, and the electric potential difference is zero. After this point of equilibrium, work would have to be done to move the next charge to B, so this would not happen by itself and would require an increase in the energy of the system. However, if you did continue to move charges, the electric potential difference from A to B would then be positive. Thus, you can see that charges would move from A to B without external forces until there is no electric potential difference between the two spheres.

Different sizes of spheres Suppose that the two spheres have different sizes, as in Figure 21-11. Although the total numbers of charges on the two spheres are the same, the larger sphere has a larger surface area, so the charges can spread farther apart, and the repulsive force between them is reduced. Thus, if the two spheres now are touched together, there will be a net force that will move charges from the smaller to the larger sphere. Again, the charges will move to the sphere with the lower electric potential until there is no electric potential difference between the two spheres. In this case, the larger sphere will have a larger charge when equilibrium is reached.

Metal Spheres of Unequal Size

Figure 21-10 A charged sphere shares charge equally with a neutral sphere of equal size when they are placed in contact with each other.

Figure 21-11 Charges are transferred from a sphere with high potential to a sphere with lower potential when they touch. The charges move to create no potential difference.
The ground wire on a fuel truck prevents ignition of the gasoline vapors. The same principle explains how charges move on the individual spheres, or on any conductor. They distribute themselves so that the net force on each charge is zero. With no force, there is no electric field along the surface of the conductor. Thus, there is no electric potential difference anywhere on the surface. The surface of a conductor is, therefore, an equipotential surface.

If a charged body is grounded by touching Earth, almost any amount of charge can flow to Earth until the electric potential difference between that body and Earth is reduced to zero. Gasoline trucks, for example, can become charged by friction. If the charge on a gasoline truck were to jump to Earth through gasoline vapor, it could cause an explosion. To prevent this, a metal wire on the truck safely conducts the charge to the ground, as shown in Figure 21-12. Similarly, if a computer is not grounded, an electric potential difference between the computer and Earth can occur. If a person then touches the computer, charges could flow through the computer to the person and damage the equipment or hurt the person.

### Electric Fields Near Conductors

The charges on a conductor are spread as far apart as they can be to make the energy of the system as low as possible. The result is that all charges are on the surface of a solid conductor. If the conductor is hollow, excess charges will move to the outer surface. If a closed metal container is charged, there will be no charges on the inside surfaces of the container. In this way, a closed metal container shields the inside from electric fields. For example, people inside a car are protected from the electric fields generated by lightning. Likewise, on an open coffee can, there will be very few charges inside and none near the bottom. Even if the inner surface of an object is pitted or bumpy, giving it a larger surface area than the outer surface, the charge still will be entirely on the outside.

On the outside of a conductor, however, the electric field often is not zero. Even though the surface of a conductor is at an equipotential, the electric field around the outside of it depends on the shape of the conductor, as well as on the electric potential difference between it and Earth. The charges are closer together at sharp points of a conductor, as indicated in Figure 21-13. Therefore, the field lines are closer together and the field is stronger. This field can become so strong that when electrons are knocked off of atoms by passing cosmic rays, the electrons and resulting ions are accelerated by the field, causing them to strike other atoms, resulting in more ionization of atoms. This chain reaction is what results in the pink glow,
such as that seen inside a gas-discharge sphere. If the field is strong enough, when the particles hit other molecules they will produce a stream of ions and electrons that form a plasma, which is a conductor. The result is a spark, or, in extreme cases, lightning. To reduce discharges and sparking, conductors that are highly charged or that operate at high potentials are made smooth in shape to reduce the electric fields.

In contrast, a lightning rod is pointed so that the electric field will be strong near the end of the rod. As the field accelerates electrons and ions, they form the start of a conducting path from the rod to the clouds. As a result of the rod’s sharply pointed shape, charges in the clouds spark to the rod, rather than to a chimney or other high point on a house or other building. From the rod, a conductor takes the charges safely to the ground.

Lightning usually requires a potential difference of millions of volts between Earth and the clouds. Even a small gas-discharge tube operates at several thousand volts. Household wiring, on the other hand, does not normally carry a high enough potential difference to cause such discharges.

**Storing Charges: The Capacitor**

When you lift a book, you increase its gravitational potential energy. This can be interpreted as storing energy in a gravitational field. In a similar way, you can store energy in an electric field. In 1746, Dutch physician and physicist Pieter Van Musschenbroek invented a small device that could store a large electric charge. In honor of the city in which he worked, it was called a Leyden jar. Benjamin Franklin used a Leyden jar to store the charge from lightning and in many other experiments. A version of the Leyden jar is still in use today in electric equipment. This new device for storing a charge has a new form, is much smaller in size, and is called a capacitor.

As charge is added to an object, the electric potential difference between that object and Earth increases. For a given shape and size of an object, the ratio of charge stored to electric potential difference, \( \frac{q}{\Delta V} \), is a constant called the capacitance, \( C \). For a small sphere far from the ground, even a small amount of added charge will increase the electric potential difference. Thus, \( C \) is small. A larger sphere can hold more charge for the same increase in electric potential difference, and its capacitance is larger.

Capacitors are designed to have specific capacitances. All capacitors are made up of two conductors that are separated by an insulator. The two conductors have equal and opposite charges. Capacitors are used today in electric circuits to store charge. Commercial capacitors, such as those shown in Figure 21-14, typically contain strips of aluminum foil separated by thin plastic that are tightly rolled up to conserve space.

The capacitance of a capacitor is independent of the charge on it, and can be measured by first placing charge \( q \) on one plate and charge \(-q\) on the other, and then measuring the electric potential difference, \( \Delta V \), that results. The capacitance is found by using the following equation, and is measured in farads, \( F \).

\[
\text{Capacitance } C = \frac{q}{\Delta V}
\]

Capacitance is the ratio of charge on one plate to potential difference.
Finding Capacitance

A sphere has an electric potential difference between it and Earth of 40.0 V when it has been charged to $2.4 \times 10^{-6}$ C. What is its capacitance?

1 Analyze and Sketch the Problem

- Draw a sphere above Earth and label the charge and potential difference.

   **Known:**  
   - $\Delta V = 40.0$ V  
   - $q = 2.4 \times 10^{-6}$ C

   **Unknown:**  
   - $C = ?$

2 Solve for the Unknown

$$C = \frac{q}{\Delta V}$$

$$= \frac{2.4 \times 10^{-6} \text{ C}}{40.0 \text{ V}}$$

$$= 6.0 \times 10^{-8} \text{ F}$$

$$= 0.060 \mu \text{F}$$

3 Evaluate the Answer

- **Are the units correct?** $\frac{\text{C}}{\text{V}} = \text{F}$. The units are farads.
- **Is the magnitude realistic?** A small capacitance would store a small charge at a low voltage.
35. **Potential Difference** What is the difference between electric potential energy and electric potential difference?

36. **Electric Field and Potential Difference** Show that a volt per meter is the same as a newton per coulomb.

37. **Millikan Experiment** When the charge on an oil drop suspended in a Millikan apparatus is changed, the drop begins to fall. How should the potential difference on the plates be changed to bring the drop back into balance?

38. **Charge and Potential Difference** In problem 37, if changing the potential difference has no effect on the falling drop, what does this tell you about the new charge on the drop?

39. **Capacitance** How much charge is stored on a 0.47-µF capacitor when a potential difference of 12 V is applied to it?

40. **Charge Sharing** If a large, positively charged, conducting sphere is touched by a small, negatively charged, conducting sphere, what can be said about the following?

   a. the potentials of the two spheres
   b. the charges on the two spheres

41. **Critical Thinking** Referring back to Figure 21-3a, explain how charge continues to build up on the metal dome of a Van de Graaff generator. In particular, why isn’t charge repelled back onto the belt at point B?

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**Varieties of Capacitors** Capacitors have many shapes and sizes, as shown in Figure 21-14. Some are large enough to fill whole rooms and can store enough charge to create artificial lightning or power giant lasers that release thousands of joules of energy in a few billionths of a second. Capacitors in television sets can store enough charge at several hundred volts to be very dangerous if they are touched. These capacitors can remain charged for hours after the televisions have been turned off. This is why you should not open the case of a television or a computer monitor even if it is unplugged.

The capacitance of a capacitor is controlled by varying the surface area of the two conductors, or plates, within a capacitor, by the distance between the plates, and by the nature of the insulating material. Capacitors are named for the type of insulator, or dielectric, used to separate the plates, and include ceramic, mica, polyester, paper, and air. Higher capacitance is obtained by increasing the surface area and decreasing the separation of the plates. Certain dielectrics have the ability to effectively offset some of the charge on the plates and allow more charge to be stored.