

SIGNIFICANT FIGURES

There are three basic rules to determine the number of significant figures in a number and conventions that govern the significant figures in a calculation (i.e. addition/subtraction or multiplication/division). The three rules to determine the significant figures are:

- 1. NON-ZERO DIGITS (I.E. 1–9) ARE ALWAYS SIGNIFICANT.**
- 2. ZERO IS SIGNIFICANT WHEN IT IS BETWEEN TWO NON-ZERO DIGITS.**
- 3. ONLY THE FINAL ZERO OR TRAILING ZEROS IN THE DECIMAL PORTION OF A NUMBER ARE SIGNIFICANT.**
 - A terminal zero to the right of a decimal point in a number greater than one is significant.
Example: The quantities 279.0, 27.90 and 2.790 all contain 4 significant figures.
 - A terminal zero to the right of a decimal point in a number less than one is significant.
Example: The values 0.2790 and 0.27900 have 4 and 5 significant figures respectively.
 - A zero used to fix a decimal point in a number less than one is not significant.
Example: The quantities 0.456, 0.0456 and 0.00456 all contain 3 significant figures.

Focus on these rules and learn them well. They will be used extensively throughout the remainder of this lab course. You are well advised to do as many problems as needed to nail down the concept. Not all of the digits have meaning (significance) and, therefore, should not be written down. In science, only the numbers that have significance (derived from measurement) are written.

Rule 1: Non-zero digits are always significant.

This rule should be rather obvious. If you measure something and the device you use (say a ruler, thermometer, or scale, etc.) returns a number to you, then you have made a measurement decision and that act of measuring gives significance to that particular numeral (or digit) in the overall value you obtain.

If the mass of an object is measured as 12.3 g, this means that the mass is known to lie between 12.2 and 12.4 g. There are 3 significant figures in 12.3.

If the mass of an object is measured as 12.3456 g, this means that the mass is known to lie between 12.3455 and 12.3457 g. There are 6 significant figures in 12.3456.

A number like 26.38 would have four significant figures and 7.94 would have three.

Rule 2: Zero is significant when it is between two non-zero digits.

Suppose you had a number like 406. By the first rule, the 4 and the 6 are significant. However, to make a measurement decision on the 4 (in the hundred's place) and the 6 (in the unit's place), you must make a decision on the ten's place. Therefore, the number 406 has 3 significant figures.

The quantities 306, 30.6, 3.06 and 0.306 all contain 3 significant figures since the 0 between the 3 and 6 is significant.

Rule 3: Only the final zero or trailing zeros in the decimal portion of a number are significant.

When a number like 0.00500 is written then the very first zero (to the left of the decimal point) is put there by convention. Its sole function is to communicate unambiguously that the decimal point is a decimal point. If the number were written like this, .00500, there is a possibility that the decimal point might be mistaken for a period. Many students omit that zero. They should not.

This rule causes the most difficulty. Here are two examples of this rule where the significant zeros are in boldface:

0.00**500** 3 significant figures 0.030**40** 4 significant figures

The zeros between the decimal point and the non-zero digits serve only as space holders. They are there to put the decimal point in its correct location. They DO NOT involve measurement decisions. Thus they are not significant. Upon writing the numbers in scientific notation (5.00×10^{-3} and 3.040×10^{-2}), the non-significant zeros disappear.

Here are two more examples where the significant zeros are in boldface:

2.30×10^{-5} 3 significant figures 4.500×10^{12} 4 significant figures

Trailing zeros in a whole number.

The number 200 is considered to have only ONE significant figure while 25,000 has two.

This is based on the way each number is written. When whole numbers are written as above, the zeros, by definition, did not require a measurement decision, thus they are not significant.

However, it is entirely possible that the value 200 really does have two or three significant figures. If it does, it will be written in a different manner than 200. Typically, scientific notation is used for this purpose. If 200 has two significant figures, then 2.0×10^2 is used. If it has three, then 2.00×10^2 is used. If it had four, then 200.0 is sufficient.

Leading zeros in a whole number.

00250 has two significant figure and 005.00×10^{-4} has three.

EXACT NUMBERS

Exact numbers, such as the number of people in a room, have an infinite number of significant figures. Exact numbers are counting up how many of something are present; they are not measurements made with instruments. Another example of this are defined numbers, such as 1 foot = 12 inches. There are exactly 12 inches in one foot. Therefore, if a number is exact, it DOES NOT affect the accuracy of a calculation or the precision of the expression. Some more examples:

There are 100 years in a century.

2 molecules of hydrogen react with 1 molecule of oxygen to form 2 molecules of water.

There are 500 sheets of paper in one ream.

Interestingly, the speed of light is now a defined quantity. By definition, the value is 299,792,458 meters per second.

PRACTICE PROBLEMS

Identify the number of significant figures:

1. 3.0800 - five significant figures.
2. 0.00418 - three significant figures
3. 7.09×10^{-5} - three significant figures
4. 91,600 - three significant figures
5. 0.003005 - four significant figures
6. 3.200×10^9 - four significant figures
7. 250 - two significant figures
8. 780,000,000 - two significant figures
9. 0.0101 - three significant figures
10. 0.00800 - three significant figures

MATHAMATICAL OPERATIONS WITH SIGNIFICANT FIGURES

Products and Quotients

The following rule or convention is used for expressing a computed product or quotient with the proper number of significant digits:

THE PRODUCT OR QUOTIENT WILL BE REPORTED AS HAVING AS MANY SIGNIFICANT DIGITS AS THE NUMBER INVOLVED IN THE OPERATION WITH THE LEAST NUMBER OF SIGNIFICANT DIGITS.

For example: 0.000170×100.40

The product could be expressed with no more than three significant digits since 0.000170 has only three significant digits, and 100.40 has five. So according to the rule the product answer could only be expressed with three significant digits.

Sums and Differences

The rule for expressing a sum or difference is considerably different than the one for multiplication or division. The rule expresses that the answer should have so many digits to the right of the decimal instead of focusing in on the total number of significant digits as the other rule did. Again, the focus is upon the number of positions to the RIGHT of the decimal. This rule states:

THE SUM OR DIFFERENCE CAN BE NO MORE PRECISE THAN THE LEAST PRECISE NUMBER INVOLVED IN THE MATHEMATICAL OPERATION.

In other words the answer can have no more numbers to the right of the decimal than the number involved in the operation with the least number of positions to the right of its decimal. Precision has to do with the number of positions to the RIGHT of the decimal. The more positions to the right of the decimal, the more precise the number. So a sum or difference can have no more indicated positions to the right of the decimal as the number involved in the operation with the LEAST indicated positions to the right of it's decimal.

For example:

$$160.45 + 6.732 = 167.182 \rightarrow \mathbf{167.18}$$

The answer could be expressed to the nearest hundredths position since 160.45 is the least precise.

Other examples include:

$$45.621 + 4.3 - 6.41 = 43.5$$

and

$$17.142 + 2.0013 + 24.11 = 43.25$$

RULES FOR ROUNDING

One other topic that needs to be covered is how do we round off non-essential digits according to these rules. Although the rules should be well known at this level in your education they will be presented for completeness. The set of conventional rules for rounding include:

- 1. Determine what the last reported digit should be, according to the significant figure rules presented above.**
- 2. Consider the digit to the right of the last reported digit.**
- 3. If the digit to the right of the last reported digit is less than 5 then leave the last reported digit unchanged and discard all digits to the right.**
- 4. If the digit to the right of the last reported digit is greater than 5 then increase the last reported digit by one and discard all digits to its right.**
- 5. If the digit to the right of the last reported digit exactly 5 then add one to the reported digit if the reported digit is an odd number, and leave it unchanged if it is even.**